Design and Analysis of MEMS Gyroscopes

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What is a Gyroscope?

- Sensor that measures the angle or rate of rotation

**Spinning Gyroscopes**
- Conservation of angular momentum

**Optical Gyroscopes**
- Sagnac Effect

**Vibratory Gyroscopes**
- Coriolis Effect

**NMR Gyroscopes**
- Larmor Precession Rate

MEMS
Applications of MEMS Gyroscopes

Automotive: Reliability
- Anti-skid control

Industrial: Robustness
- Antenna stabilization
- Precision machinery

Consumer: Size & Cost
- Gaming
- Health and fitness
- Optical Image Stabilization

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IEEE SENSORS 2013
Evolution of MEMS Gyroscopes

- STMicroelectronics 3-Axis Gyroscope (Consumer)
  
  **(2009)**
  
  Package: 7.5x4.4x1.1 mm³
  
  MEMS: ~ 9.3 mm²
  
  
  **(2010)**
  
  Package: 5x4x1.1 mm³
  
  **(2011)**
  
  Package: 3x3.5x1 mm³
  
  **(2013)**
  
  Package: 3x3x1 mm³
  
- Invensense 3-Axis Gyroscope (Commercial)
  
  **(2012)**
  
  Package: 4x4x0.9 mm³
  
  **(2013)**
  
  L3GD20H
  
Performance in Gyroscopes (Consumer)

- Current applications do not demand low-noise performance
- Pedestrian and in-doors navigation → LOW NOISE IS A MUST!

Is it time for a new technology?

Noise performance tapering?
Operation Principles - The Coriolis Effect

- Example: The Foucault Pendulum

\[ \ddot{a}_{\text{cor}} = -2 \vec{\Omega} \times \vec{v}_{\text{drv}} \]

- For an extraterrestrial observer: pendulum swings back and forth
- For a terrestrial observer: Trajectory of swing changes by \( \ddot{a}_{\text{cor}} \)
Micromechanical Gyroscopes

- Example: The Tuning Fork Gyroscope (TFG)

- Equations of motion of an ideal gyroscope:

  **Mode 1**
  \[
  m \frac{\partial^2 x}{\partial t^2} + b_x \frac{\partial x}{\partial t} + k_x x = F_{elecx} + 2 m \lambda \Omega_Z \frac{\partial y}{\partial t}
  \]

  **Mode 2**
  \[
  m \frac{\partial^2 y}{\partial t^2} + b_y \frac{\partial y}{\partial t} + k_y y = F_{elecy} - 2 m \lambda \Omega_Z \frac{\partial x}{\partial t}
  \]
Modes of Operation

**Rotation-Rate Gyros**
- Output proportional to $\Omega$
- Mode 1 driven into oscillation
- Mode 2 used to detect rotation

**Whole-Angle Mode Gyros**
- Output proportional to $\theta$
- Free-vibrating structure
- Standing-wave precesses
Vibratory Rotation-Rate Gyroscopes

- Two second-order systems
  - Drive (excited into oscillation)
  - Sense (response proportional to rotation-rate $\Omega$)

\[ m \frac{\partial^2 x}{\partial t^2} + b_x \frac{\partial x}{\partial t} + k_x x = F_{elecx} \]

\[ m \frac{\partial^2 y}{\partial t^2} + b_y \frac{\partial y}{\partial t} + k_y y = -2m \lambda \Omega \frac{\partial x}{\partial t} \]

\[ \vec{F}_{cor} = -2m \lambda \Omega \frac{\partial x}{\partial t} \]
Driving the Gyroscope

• To generate $v_{drv}$, one mode is driven (usually into oscillation)

\[ m \frac{\partial^2 x}{\partial t^2} + b_x \frac{\partial x}{\partial t} + k_x x = F_{elecx} \]

• Frequency-domain:

\[ \frac{X(j\omega)}{F_{elec}(j\omega)} = \frac{1}{m} \left( \frac{1}{-\omega^2 + \frac{\omega_{0drv}^2}{Q_{drv}} j\omega + \omega_{0drv}^2} \right) \]

• At resonance ($\omega = \omega_{0drv}$):

\[ \left| \frac{X(j\omega)}{F_{elecx}(j\omega)} \right| = \frac{Q_{drv}}{m \omega_{0drv}^2} \text{ and } \angle \frac{X(j\omega)}{F_{elecx}(j\omega)} = -90^\circ \]
Electrostatic Transducers

Parallel-Plate Transducer

- High electromechanical coupling
- Small and easy to implement
- Non-linear transfer function

\[
\frac{dC}{dx} = \frac{\varepsilon \cdot w \cdot t}{(g_0 - x)^2} \approx \frac{\varepsilon \cdot w \cdot t}{g_0^2}
\]

Comb-Drive Actuation

- Linear actuation
- Allows large displacements
- Low coupling coefficient

\[
\frac{dC}{dx} = \frac{\varepsilon \cdot 2n \cdot t}{g_0}
\]
Detecting Rotation Rate

- With $\nu_{drv}$ established, the sense mode responds in presence of $\Omega$

\[
m \frac{\partial^2 y}{\partial t^2} + b_y \frac{\partial y}{\partial t} + k_y y = F_{cor}
\]
\[
F_{cor} = 2m \lambda \Omega z \frac{\partial x}{\partial t}
\]

- Frequency-domain:

\[
\left. \frac{Y(j\omega)}{X(j\omega)} \right|_{\omega = \omega_{0,drv}} = 2 \lambda \Omega \frac{j\omega_{0,drv}}{-\omega_{0,drv}^2 + \frac{\omega_{0,sns}^2}{Q_{sns}} \left( j\omega_{0,drv} + \omega_{0,sns} \right)}
\]

- But where is $\omega_{0,sns}$ with respect to $\omega_{0,drv}$?
Rate Gyros - Modes of Operation

- Mode-Split: Drive and sense frequencies are different
- Mode-Matched: Drive and sense frequencies are identical

**If** $\omega_{0_{drv}} << \omega_{0_{sns}}$:

$$\left| \frac{y}{x} \right|_{split} \approx 2 \lambda \Omega_z \frac{\omega_{0_{drv}}}{\omega_{0_{sns}}^2}$$

**If** $\omega_{0_{drv}} = \omega_{0_{sns}}$:

$$\left| \frac{y}{x} \right|_{matched} = 2 \lambda \Omega_z \frac{Q}{\omega_0}$$
Mode-Split vs. Mode-Matched Gyros

**Mode-Split Gyros**
- Typically of Tuning-Fork kind
- Modes from different mechanisms
- Large BW (accelerometer response)
- Scale factor $\propto 1/\omega^2_{\text{sns}}$
  - Large mass (bigger size)
  - Low spring constant (poor reliability)

**Mode-Matched Gyros**
- Typically axisymmetric
- Inherent degenerate modes
- BW proportional to $f_0/Q$
- Scale factor $\propto Q$
  - 10,000 to 1’000,000 larger!!
Mode-Split Rate Gyroscopes

- Typically TFGs $\rightarrow$ Low resonance frequency (1 – 30 kHz)

$$SF \propto \frac{x_{drv} \omega_{0drv}}{\sqrt{\left(\omega_{0sns}^2 - \omega_{0drv}^2\right)^2 + \frac{\omega_{0sns}^2 \omega_{0drv}^2}{Q_{sns}}}} \frac{dC}{dx} V_p$$

- To compensate for loss of Q-amplification:
  - Larger mass
  - Lower stiffness
  - Interdigitated and comb capacitors

- For large $x_{drv}$, high-Q still needed on drive
- In kHz range, high-vacuum required for high Q $\rightarrow$ GETTERS
Bulk-Acoustic Wave (BAW) Gyroscopes

- Axisymmetric structure $\rightarrow$ Inherently mode-matched
- $Q = 50,000$ to $200,000$ in 1 to 10 Torr $\rightarrow$ High sensitivity, low noise
- High $f_0$ (MHz range) $\rightarrow$ Large BW, dynamic range, shock resistance

- Capacitive nano-gaps
  - Large electromechanical coupling

H. Johari et al, MEMS, 2007
Operation BAW Rate Gyroscopes

$n = 3$ degenerate modes
Implementation of BAW Gyroscopes

9.6 MHz BAW Disk Gyro

HARPSS™ Process

Wafer-level Package

High Q at 1 – 10 Torr

Frequency Response - Mode-Matched Gyro

Q = 77,000
Performance of Capacitive BAW Gyros

**Motional Impedance**

\[ R_m = \frac{2\pi \cdot M_{\text{eff}} \cdot g_0^4 \cdot f_{\text{res}}}{(\varepsilon_0 \cdot A_{\text{elec}} \cdot V_P)^2 \cdot Q} \quad [\Omega] \]

**Scale Factor**

\[ SF = \frac{2\pi \cdot \lambda \cdot \varepsilon_0 \cdot A_{\text{elec}} \cdot V_P \cdot Q}{180 \cdot \alpha \cdot g_0} \quad [A/(s/\circ)] \]

**Mechanical Noise**

\[ MNE\Omega = \frac{180 \cdot \alpha}{\pi \cdot \lambda \cdot g_0} \sqrt{\frac{k_B \cdot T}{\pi \cdot M_{\text{eff}} \cdot f_{\text{res}} \cdot Q}} \quad [/(s/\circ)^{1/2}\text{Hz}] \]

**Bandwidth**

\[ BW = \frac{f_{\text{res}}}{2Q} \quad [\text{Hz}] \]

- Lower is better!
  - High \( Q \) (~50,000 @ 1 – 10 Torr)
  - Ultra-small capacitive nano-gaps

- Higher is better!
  - Independent of frequency!!

- Lower is better!
  - High \( f_{\text{res}} \) & high \( Q \) compensate for smaller displacements

- Higher is better!
  - High \( f_{\text{res}} \) compensates for high \( Q \)
Pitch and Roll Annulus Gyroscopes

- High frequency operation (0.5 ~ 1.5 MHz)
- Process compatible with HARPSS™ → air nano-gaps

Out-of-Plane HARPSS™ capacitive nano-gaps

In-plane drive mode

Out-of-plane x-axis sense mode

W. K. Sung et al, TRANSDUCERS, 2011

Out-of-plane y-axis sense mode
Annulus Gyroscopes - Response

- Small frequency split (further compensated with electronics)

Δf ≈ 300 Hz @ 0.5 MHz

Rate Response

W. K. Sung et al, TRANSUCERS, 2011
Multi-Degree-of-Freedom Integration

Monolithic TIMU Die

WLP TIMU Die

X-Axis Accelerometer

X-Axis Gyroscope

Z-Axis Accelerometer

Z-Axis Gyroscope

Y-Axis Accelerometer

Y-Axis Gyroscope

X-Axis Axial Response

Z-Axis Axial Response

Y-Axis Axial Response

Resonator

X-Axis Gyro Response

Y-Axis Gyro Response

Z-Axis Gyro Response

Q ≈ 28,000

Q ≈ 28,000

Q ≈ 118,000
Error Sources in Mode-Matched Gyros

- **Mode 1:**

\[
m_{11} \ddot{q}_1(t) + d_{11} \dot{q}_1(t) + k_{11} q_1(t) = -2\lambda m_{22} \dot{q}_2(t)\Omega(t)
\]

- **Mode 2:**

\[
m_{22} \ddot{q}_2(t) + d_{22} \dot{q}_2(t) + k_{22} q_2(t) = 2\lambda m_{11} \dot{q}_1(t)\Omega(t)
\]

- **Ideal gyroscope:**

\[
\omega_{01} = \sqrt{\frac{k_{11}}{m_{11}}} = \omega_{02} = \sqrt{\frac{k_{22}}{m_{22}}}
\]

- **Anisoelasticity:**

\[
k_{22} \neq k_{11}
\]

- **Anisoinertia:**

\[
m_{22} \neq m_{11}
\]
Compensating for Frequency-Split

- Electrostatic Spring Softening

\[ \omega_{01} = \frac{\sqrt{k_{11\text{mech}} - k_{11\text{elec}}}}{m_{11}} \]

\[ k_{11\text{elec}} = \frac{\varepsilon A}{s_0^3} \sum_{j=1}^{l} (V_P - V_{T,j})^2 \]

\[ \omega_0 = \sqrt{\frac{k_{11\text{mech}} - k_{11\text{elec}}}{m_{11}}} \]

Freqency Tuning of Aligned Gyro

- Spring softening

Aligned with mode 1

\[ V_T = V_P \]

\[ V_T = 7\, V \]
Mode-to-Mode Coupling

- Ideal gyroscope with uncoupled modes

- Imperfect gyroscope with coupled modes

\[ m \ddot{q}_1(t) + d_{11} \dot{q}_1(t) + d_{12} \dot{q}_2(t) + k_{11} q_1(t) + k_{12} q_2(t) = -2 \lambda m \dot{q}_2(t) \Omega(t) \]

\[ m \ddot{q}_2(t) + d_{22} \dot{q}_2(t) + d_{21} \dot{q}_1(t) + k_{22} q_2(t) + k_{21} q_1(t) = 2 \lambda m \dot{q}_1(t) \Omega(t) \]
Stiffness Coupling

- Mode 1 displacement generates force that couples to Mode 2

![Diagram showing coupling between Mode 1 and Mode 2](image)

If $k_{11} = k_{22}$ (i.e. $\Delta \omega$ close to 0):

$$\angle \frac{q_2}{q_1} \approx -90^\circ$$

Quadrature

- Electrostatic mode-decoupling:

  - @ $V_Q = V_{QA} \rightarrow q_{2Q} = 0$ (modes decoupled)
Damping Coupling

- Mode 1 velocity generates force that couples to Mode 2

\[
\ddot{q}_{21}(t) + \frac{\omega_0^2}{Q_2} \dot{q}_{21}(t) + \frac{b_{21}}{m} \dot{q}_1(t) + \omega_0^2 q_{21}(t) = 0
\]

- For a mode-matched gyroscope:

\[
\left. \frac{q_{21}(\omega)}{q_1(\omega)} \right|_{\omega=\omega_01=\omega_02} = -\frac{b_{21} Q_2}{m \omega_01} \angle 0^0
\]

- Comparing with Coriolis coupling due to rotation rate

\[
\ddot{q}_{2c}(t) + \frac{\omega_0^2}{Q_2} \dot{q}_{2c}(t) + \omega_0^2 q_{2c}(t) = 2\lambda \Omega(t) \dot{q}_1(t)
\]

- For a mode-matched gyroscope:

\[
\left. \frac{q_{2c}(\omega)}{q_1(\omega)} \right|_{\omega=\omega_01=\omega_02} = \frac{2\lambda Q_2}{\omega_01} \Omega(\omega') \angle 0^0
\]

\[q_{21} \text{ is indistinguishable from } q_{2c}\]
Loss Mechanisms in Resonant Gyros

- Q in resonant gyroscopes is a combination of different effects:

\[ \frac{1}{Q} = \frac{1}{Q_{SFD}} + \frac{1}{Q_{TED}} + \frac{1}{Q_{anchor}} + \frac{1}{Q_{surface}} + \frac{1}{Q_{intrinsic}} \]

**Squeeze-Film Damping**

\[ \frac{1}{Q_{SFD}} = \alpha \frac{\mu_{eff}}{g_0^3} \frac{1}{1 + j\omega/\omega_c} \]

Higher Q at higher freq.

**Thermoelastic Damping**

\[ \frac{1}{Q_{TED}} = \left( \frac{E \alpha^2 T_0}{C_v} \right) \sum_n \frac{\omega_{mech} \tau_n}{1 + (\omega_{mech} \tau_n)^2} f_n \]

Depends on CTE and therm. modes

**Anchor Loss**

\[ \frac{1}{Q_{anchor}} = \frac{1}{2\pi} \frac{W_{resonator}}{\Delta W_{anchor}} \]

SED @ anchor
Piezoelectric Square Gyroscope

- Capacitive transducers, well established in sensors, but:
  - Low electromechanical coupling coefficients
  - Non-linear (Parallel plate)
- Piezoelectric transduction → widely used in resonators

R. Tabrizian, et. al., JMEMS, 2013

Lamb-Wave Mode Pairs
Whole-Angle Mode Gyroscopes

- Also known as rate-integrating gyroscopes (RIG)
- Strap-down navigation utilizes angle and displacement information

Integration step introduces error and accumulates drift
- Whole-angle mode → output proportional to angle, not rate
Operation of Whole-Angle Mode Gyros

- Based on relative measurement with respect to a standing-wave
- Similar to the Foucault Pendulum example

**Angular gain**

\[
\lambda = \frac{27}{90} \approx 0.3
\]

- Anti-nodes precess with respect to reference frame
- The angular gain factor \(\rightarrow\) very stable parameter
But Why Precession?

- First discovered by G.H. Bryan (circa 1890)

- Nodes $\rightarrow$ no radial component (i.e. no Coriolis effect)
- Antinodes $\rightarrow$ Maximum radial displacement (i.e. max Coriolis)
- Thus, antinodes have to rotate much faster than nodes
Detection of Rotation Angle

- Breaking down the vibration into orthogonal components:
  \[ \frac{q_2}{q_1} = \tan 2\theta \]

- \( \theta \) is the pattern angle \( \rightarrow \) Related to rotation angle through \( \lambda \)

- Two sets of differential electrodes
  - Cosine electrodes: \( q_1 \cdot \cos(\omega_0 t) \)
  - Sine electrodes \( q_2 \cdot \sin(\omega_0 t) \)

- \( q_1 \) and \( q_2 \) obtained by demodulation
- arctan of their ratio \( \rightarrow \theta \)
Limitations of MEMS Whole-Angle Gyros

- To operate, pattern of vibration should not be perturbed
- But, amplitude of vibration decays with time $\Rightarrow$ limited Q

- Long decay times needed
- High Q, low frequency
  - MEMS hemispherical resonator gyros
Summary

- Improvements in resolution still required for personal navigation
- Shift in design methodology is imminent to achieve performance
- Vibration and shock immunity are more important than thought
- High-frequency BAW gyros:
  - Rugged structures with clear advantages over TFG designs
  - Easy to integrate into monolithic multi-DOF units
- Whole-angle MEMS gyros → plenty of room for improvement
References

References


